On the equivalence between welfarism and equality of opportunity

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Abstract

A welfarist way of allocating resources consists in 1) equipping individuals with comparable indices of their well-being and 2) applying a unique aggregation rule to individual well-being levels. An equality of opportunity way of allocating resources consists in 1) making the distinction between personal characteristics which are under and beyond individuals' control, and 2) decreasing inequalities due to differences in characteristics beyond individuals' control. We show that under the proviso that indifferent individuals should not influence social judgements, welfarist and equal opportunity judgements on resource allocation are equivalent.

1 Introduction

Two main ethical theories have the lead in welfare economics, welfarism and equality of opportunity. Welfarism is the view that individual utilities are all what matters for equitable decision making. Public decisions of resource allocation should all be driven by their impact on individual utilities. Once impacts on utilities are determined, collective choices are made on ground

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of an aggegation rule of individual utilities. This aggregation rule does not depend on the specific choice to be made, nor on the specific utility functions of the agents. Utilitarianism, for instance, is the welfarist theory where aggregation is obtained by summing up individual utilities.

Welfarism is prevalent, for instance, in public economics, where optimal taxation theory is built under the assumption that the social planner tries to maximize a welfarist (usually utilitarian) social welfare function. There are two main criticisms addressed towards welfarism. First, political philosophers object against it that social justice cannot be stated in terms of individual utilities. Rawls, in particular, has forcefully argued that taking utilities into account conflicts with a view of human beings as autonomous moral agents (see Rawls [20]). Second, welfarism also suffers from the fact that there is no consensus among economists (nor among psychologist or any others) on how to measure utilities. Public economists circumvent the latter difficulty by stating that individual utilities should not necessarily reflect agents' happiness or satisfaction but, rather, they reflect the ethical choices of the planner, that is, how the planner views the agents' happiness or satisfaction.

Recently, theories of justice based on the idea that opportunities should be equalized have been applied to economic issues. There are several competing theories of equality of opportunity (see below, beginning of section 3). All agree that differences in agents' outcomes come from differences in characteristics they should be responsible for (e.g. because they control the value taken by those characteristics) and differences in characteristics they should not be responsible for. Equalizing opportunities consists of allocating external, transferable resources in such a way that differences in the latter characteristics, and only those differences, are eliminated. Those theories differ in how they define outcomes, and in where they put the cut between characteristics that need to be counterbalanced, which we call here compensation parameters, and characteristics which do not justify any intervention, which we call responsibility parameters. Once outcomes are defined and the compensation/responsibility cut is chosen, theories also differ with respect to the way lists of (typically unequal) individual opportunity sets are compared, that is, how individual opportunities are aggregated (see, e.g., Roemer [22] and Bossert, Fleurbaey and Van de gaer [6], or Kranich [17], [18], and Kranich and Ok [19]).

In this paper, we show equivalence between these two ethical theories. More precisely, we prove that if a consistency condition is imposed then any social welfare judgement based on the idea that opportunities should be equalized is equivalent to building individual utility functions and applying a utility aggregation rule that only depends on the utility levels. Consistency means that removing indifferent agents does not influence the social preference over suballocations. Consistency properties have been extensively studied in game theory and the fair allocation literature (see Thomson [27]). They are also reminiscent to separability conditions which are common in welfare economics (see e. g. Fleming [11] and d'Aspremont and Gevers [8]). The major aggregation rules encountered in social choice theory satisfy consistency (like the utilitarian, leximin and Nash social welfare functions; see below, section 3).

A natural question raised by this result is that of the informational basis of those ethical theories. In particular, is it possible to construct utility functions in the way suggested here by relying only on ordinal noncomparable information on individual preferences, which is the only information revealed by deterministic individual choices? We prove it is, provided individual preferences are part of the responsibility parameters, that is, parameters for which differences among individuals do not justify any compensation.

The main achievement of this paper is therefore that it opens a way for solving the problem faced by welfarist social observers as of how to construct utility functions in an ethically maeningful way. We show, indeed, that equal opportunity requirements can be used to help perform this construction, even by sticking to traditional ordinal non-comparable preferences, consistent with revealed preferences.

The paper is organized as follows. We define the model in Section 2. We introduce and justify our properties in Section 3. We state the equivalence result in Section 4. We prove the compatibility between our approach and ordinalism non-comparability in Section 5. We give all the proofs in Section 6, and we conclude in Section 7.

2 Model and notations

We borrow the model from Fleurbaey [14]. It is the simplest model where both equality of opportunity and welfarism make sense. There are variable sets of agents drawn from the infinite set of all possible populations $\mathcal{N}$. For a set of agents $N \in \mathcal{N}$, each agent $i \in N$ is characterized by two lists of parameters $(\theta_{ic}, \theta_{ir}) \in \Theta_c \times \Theta_r$. By convention, the list θ_{ic} refers to the parameters for which the society would like to compensate the agent, whereas the list θ_{ir} refers to the parameters the value of which the agent should be held responsible for.

The characterization of agents through two sets of parameters is necessary to introduce equality of opportunity allocation ordering functions. Being able to vary the value of these characteristics is necessary to define welfarism, and we have chosen to vary those values by allowing the population to change, so that an agent can be replaced by another agent with other characteristics.

Agents are likely to obtain amounts of resources. The set of possible resources amounts is denoted X. An element $x_i \in X$ can be interpreted as a bundle of goods, or, preferably, as a budget (or opportunity) set. Given resources $x_i \in X$ and parameters $(\theta_{ci}, \theta_{ri}) \in \Theta_c \times \Theta_r$, agent i reaches outcome $O(x_i, \theta_{ci}, \theta_{ri}) \in \mathcal{O}$. It is sufficient for our purpose to assume that $\mathcal{O}$ is a partially ordered set. On the other hand, let us observe that function $O : X \times \Theta_c \times \Theta_r \to \mathcal{O}$ is not parameterized by any $i \in N$. This is consistent with the fact that any parameter determining the relevant characteristics of the agents are embodied in the θ's.

Let us give two examples. Assume first that we wish to equalize opportunities in the schooling system (such an analysis is carried out in Roemer [22]). We may consider that students are not responsible for their genes, nor for the socio-economic backgrounds of their parents, whereas they should be accountable for their schooling efforts. In this case, θ_{ic} represents student i's genes and her parents' characteristics, θ_{ir} represents her schooling effort, measured, say, in hours of work, x_i stands for the per student expenses of the government in student i's school, and $O(x_i, \theta_{ci}, \theta_{ri})$ stands for the wage rate, or the wage rate opportunity reached by i.

Assume, as a second example, that we study income taxation on the labor market (no reform of the schooling system is possible). We may consider that workers are not responsible for their wage opportunities, whereas they are free to choose their (yearly, or lifetime) labor time (see the analyses in Bossert [5], Fleurbaey and Maniquet [16] Bossert, Fleurbaey and Van de gaer [6] and Sprumont [25]). Then we would have θ_{ic} for the wage rate, θ_{ir} for the labor time, x_i for the tax paid or subsidy received, and $O(x_i, \theta_{ci}, \theta_{ri})$ for (an index of) the consumption level.

An *economy* is a set of agents $N \in \mathcal{N}$, and a list of individual characteristics. Formally, $e = (N, \theta_c, \theta_r) \in \mathcal{E} = \mathcal{N} \times \cup_{N \in N} \Theta_c^N \times \Theta_r^N$. An *allocation for an economy* $e = (N, \theta_c, \theta_r) \in \mathcal{E}$ is a list $x \in X^N$ of individual resource bundles. For an economy $e = (N, \theta_c, \theta_r) \in \mathcal{E}$, the problem faced by the ethical planner is to order elements of X^N as a function of the characteristics of the agents. As the set of agents is variable, an ethical theory can be represented by a resource allocation ordering function $\overline{R}$ whose domain is the set of economies, $\mathcal{E}$, and such that for all $e \in \mathcal{E}$, $\overline{R}(e)$ is a complete ranking on X^N.

We will restrict ourselves in this paper to problems where compensation is possible, that is, where the amount of resources needed to equalize opportunities is always finite.

Assumption A: for all $\theta_{ci}, \theta'_{ci} \in \Theta_c, \theta_{ri} \in \Theta_r, x_i \in X$, there exists $x' \in X$ such that $O\left(x_i, \theta_{ci}, \theta_{ri}\right) = O\left(x'_i, \theta'_{ci}, \theta_{ri}\right)$.

Even if this is a severe limitation (it is indeed likely that no finite amount of money would lead any human in good health agree to become heavily handicapped), it is legitimate to raise the question of compensation only when nature does not exclude by itself the possibility of compensating (as it is the case, for instance, in the two examples above).

3 Properties

In this Section, we give the formal definitions of the ethical theories we are interested in. Our definitions will be axiomatic. We will begin with equality of opportunity, which is not as well known among economists as welfarism.

"There is, in the notion of equality of opportunity, a 'before' and an 'after': before the competition starts, opportunities must be equalized, by social intervention if need be, but after it begins, individuals are on their own" (Roemer [22], p2). Let us call the 'before' principle of compensation, and the 'after' principle of responsibility.

There are three branches of economic literature on equality of opportunity. They differ in several respects. First, they do not all give the same emphasis on the responsibility principle. Second, they do not use the same method of justification to their proposals. Some are axiomatic, some are not. Third, the extent to which they have led to applications varies from one another.

The first branch of literature, initiated by Roemer [21], directly addresses the question of the definition of the social optimum, in the social welfare function tradition. Another key feature of the approach is that the part of an individual's outcome for which she is responsible is defined as her precise place in the statistical distribution of outcomes among agents of the same type (see [28]). This approach has led to various applications, studying for instance how to finance the schooling system or the health system (see Roemer [22]).

The second branch, initiated by Fleurbaey [12] and Bossert [5], tries to define the social optimum axiomatically, and focusses on the possible dilemma between the principles of compensation and responsibility. The approach has been applied to health care insurance system (see Schokkaert, Dhaene and Van de Voorde [23]) and minimum income (see Fleurbaey, Hagnere, Martinez and Trannoy [15]).

The third one, initiated by Kranich [17], concentrates on the compensation principle, and axiomatically develops ways to measure the degrees of achievements of the compensation goals. To the best of our knowledge, it has not yet given rise to empirical applications.

We begin by stating properties which allocation ordering functions should satisfy if they are to capture the equality of opportunity ethics. Given the different theoretical developments of these ideas in the economic literature, our strategy is to define weak properties which may pretend to be at the intersection of all three main branches of the literature. All those properties are inspired by the indifference version of Suppes' grading principles (Suppes [26], Sen [24]), and, therefore, they are consistent with any degree of inequality aversion. The first property, called Compensation, requires that permuting the outcome levels of two agents having the same responsibility parameters but possibly different compensation parameters leads to two socially equivalent allocations. The justification is clear: as those agents have the same responsibility parameters, society should treat the outcomes of those two agents anonymously; if outcomes are permuted, then we obtain an equally good, or equally bad, allocation.

Compensation: for all $e = (N, \theta_c, \theta_r) \in \mathcal{E}$, $j, k \in N$ such that $\theta_{rj} = \theta_{rk}$, $x', x' \in X^N$ such that $x_i = x'_i$ for all $i \neq j, k$: if $O(x_j, \theta_{cj}, \theta_{rj}) = O(x'_k, \theta_{ck}, \theta_{rk})$ and $O(x'_j, \theta_{cj}, \theta_{rj}) = O(x_k, \theta_{ck}, \theta_{rk})$, then $x\,\overline{I}(e)\,x'$.

It will be sufficient in some cases below to focus on the following much weaker property. It offers a similar requirement as that above but restricted to the cases where the agents' responsibility parameters are equal to some reference parameter $\tilde{\theta}_r \in \Theta_r$.

Minimal Compensation: there exists $\tilde{\theta}_r \in \Theta_r$ such that for all $e = (N, \theta_c, \theta_r) \in \mathcal{E}$, $j, k \in N$ such that $\theta_{rj} = \theta_{rk} = \tilde{\theta}_r$, $x, x' \in X^N$ such that $x_i = x'_i$ for all $i \neq$

j,k: if $O(x_j,\theta_{cj},\theta_{rj}) = O(x'_k,\theta_{ck},\theta_{rk})$ and $O(x'_j,\theta_{cj},\theta_{rj}) = O(x_k,\theta_{ck},\theta_{rk})$, then $x\,\overline{I}(e)\,x'$.

The two next properties are consistent with the idea that society should counterbalance differences in compensation parameters only. That is, if two agents differ only in terms of their responsibility parameters, society should treat them indifferently, so that their respective outcomes reflect the differences in the parameters they control. The third property, called Responsibility, requires that permuting the bundles of two agents having the same compensation parameters but possibly different responsibility parameters leads to two socially equivalent allocations. Again, the justification is clear: as those agents have the same compensation parameters, society should treat the resources allocated to those two agents anonymously. By permuting resource bundles, we obtain an equally good, or equally bad, allocation.

Responsibility: for all $e = (N,\theta_c,\theta_r) \in \mathcal{E}$, $j,k \in N$ such that $\theta_{cj} = \theta_{ck}$, $x,x' \in X^N$ such that $x_i = x'_i$ for all $i \neq j,k$: if $x_j = x'_k$ and $x'_j = x_k$, then $x\,\overline{I}(e)\,x'$.

It will also prove sufficient in some cases below to focus on a weaker property, obtained by restricting the above requirement to the cases where the agents' compensation parameters are equal to some reference parameter $\tilde{\theta}_c \in \Theta_c$.

Minimal Responsibility: there exists $\tilde{\theta}_c \in \Theta_c$ such that for all $e = (N,\theta_c,\theta_r) \in \mathcal{E}$, $j,k \in N$ such that $\theta_{cj} = \theta_{ck} = \tilde{\theta}_c$, $x,x' \in X^N$ such that $x_i = x'_i$ for all $i \neq j,k$: if $x_j = x'_k$ and $x'_j = x_k$, then $x\,\overline{I}(e)\,x'$.

Resource should be allocated to equalize opportunities. Consequently, resource bundles only matter inasmuch as they allow agents to reach high outcome levels. The crucial parameters which the planner should look at to compare allocations should not be the resource bundles as such but agents' outcomes. This is captured by the following Social Indifference property. It requires that two allocations leading the same outcome to each agent should be deemed equivalent. It is reminiscent to the usual Pareto Indifference property. Recall that we did not introduce preferences explicitly in our setting.

Social Indifference: for all $e = (N,\theta_c,\theta_r) \in \mathcal{E}$, $x,x' \in X^N$, if $O(x_i,\theta_{ci},\theta_{ri}) = O(x'_i,\theta_{ci},\theta_{ri})$ for all $i \in N$, then $x\,\overline{I}(e)\,x'$.

We consider that the five axioms defined so far are the cornerstone of equality of opportunity. Any particular theory could, of course, be more demanding about either the compensation or the responsibility principle, but the ideas captured in those principles must be part of any specific theory. At the generality level of the model we study here, however, the existence of social ordering functions satisfying Compensation and Responsibility is not guaranteed (thereby illustrating the trade-off analysed in the second branch of the literature presented above), so that we will restrict ourselves to combining either principle with the minimal version of the other.

This paper is aimed at studying the relationship between recent theories of equality of opportunity and welfarism, the long since dominating theory in welfare economics. It is essential here to distinguish between two different traditions in welfarist ethics. Of course, all welfarists agree that collective choice should be a matter of welfare aggregation, but not all welfarists agree on what welfare refers to.

In the first tradition, welfare refers to some subjective appraisal of one's own well-being (see e. g., Blackorby, Bossert and Donaldson [3]). In the second tradition, welfare is not necessarily a subjective notion but can refer to any a priori comparable indices of individual well-being (see d'Aspremont and Gevers [9]). These indices are commonly interpreted as utility functions representing individual preferences, but may as well be summaries of individuals' doings and beings, or life expectancy, etc. That is, welfarism, in the latter tradition, is a flexible ethical theory, as it can be used to aggregate any kind of well-being indicators. But, as a consequence, it is an incomplete theory, as it does not tell the ethical observer how to construct those indicators. This paper is an attempt to complement this tradition with an ethically meaningful theory of how to construct welfare indicators.

We now proceed by recalling the definition of welfarism (see e.g. d'Aspremont and Gevers [9]).

A *utility function* is a function $u : X \to \mathbb{R}$. The set of all utility functions is denoted $\mathcal{U}$. A *utility representation of the outcome function* is a function $u_{..} : \Theta_c \times \Theta_r \to \mathcal{U}$ such that for all $(\theta_c, \theta_r) \in \Theta_c \times \Theta_r$, and $x, x' \in X$:

$$[O(x, \theta_c, \theta_r) \geq O(x', \theta_c, \theta_r)] \Leftrightarrow [u_{\theta_c,\theta_r}(x) \geq u_{\theta_c,\theta_r}(x')].$$

A *social welfare ordering* for $N \in \mathcal{N}$ is a complete ordering on $\mathbb{R}^N$. A *social welfare ordering function*, denoted $\underline{R}$, associates to each $N \in \mathcal{N}$ an ordering $\underline{R}(N)$ on $\mathbb{R}^N$.

Welfarism: there exist $u_{..}$ and a unique $\underline{R}$ such that for all $e = (N, \theta_c, \theta_r) \in \mathcal{E}$, $x, x' \in X^N$

$$x\, \overline{R}(e)\, x' \Leftrightarrow u\, \underline{R}(e)\, u'$$

where for all $i \in N$, $u_i = u_{\theta_{ic},\theta_{ir}}(x_i)$ and $u'_i = u_{\theta_{ic},\theta_{ir}}(x'_i)$ and $\underline{R}$ is said to be associated to $\overline{R}$.

We will restrict our attention to welfarist ordering functions that are minimally equitable in the sense of anonymity. It is clear that any defendable ethical theory satisfies this requirement. *Anonymity* requires that the names of the agents do not matter in social judgements, that is, if two agents permute their utility levels, then the social welfare ranking remains unaffected. For $N \in \mathcal{N}$, let Π_N denote the set of all permutations of N.

Anonymity$^{\text{w}}$: for all $N \in \mathcal{N}$, $u, u' \in \mathbb{R}^N$ and $\pi \in \Pi_N$: if $u \, \underline{R}(N) \, u'$ then $\pi(u) \, \underline{R}(N) \, \pi(u')$.

Our main result, stated in the following Section, establishes the equivalence between the two theories we are interested in, provided they satisfy some cross-economy consistency requirement.[1] The next axiom is a natural adaptation in our framework of the consistency property which has been extensively studied in game theory and the fair allocation literature (see Thomson [27]). Consistency works like this. If an allocation is socially as good as another, and if a subset of agents are assigned exactly the same resource bundles in both allocations, then removing those agents with their resources should not change the social preference, that is, the suballocation obtained from the first allocation by removing those agents should still be as good for the subeconomy as the allocation obtained from the second allocation.

Consistency: for all $e = (N, \theta_c, \theta_r) \in \mathcal{E}$, $M \subset N, x, x' \in X^N$, if $x_i = x'_i$ for all $i \in N \setminus M$, then

$$\begin{aligned} x \, \overline{R}\,(e)\; x' &\;\Rightarrow\; x_M \, \overline{R}\,(M, \theta_{cM}, \theta_{rM})\; x'_M, \\ x \, \overline{P}\,(e)\; x' &\;\Rightarrow\; x_M \, \overline{P}\,(M, \theta_{cM}, \theta_{rM})\; x'_M. \end{aligned}$$

Consistency properties in the same spirit as this property have been extensively studied in the equality of opportunity literature. Unfortunately, Consistency, as we define it in this paper, is not compatible with the approach proposed be Roemer, as removing agents from an economy affects the

[1] Readers familiar with the axiomatic approach may have noticed that all the equality of opportunity requirements are single profile requirements, whereas welfarism is multi-profile. A comparison is therefore possible only if some inter profile requirement is added. Consistency is such a requirement, although a very weak one.

relative place in the outcome distribution of others[2] (recall that in Roemer's approach, an agent is responsible for her place in the statistical distribution of outcomes among agents of the same type). On the other hand, there is no difficulty in combining Consistency with the other two approaches to equality of opportunity.[3]

A welfarist social welfare ordering function satisfies Consistency if and only if the social welfare ordeing function which is associated to it satisfies the following Consistency$^{\mathrm{w}}$ axiom.

Consistency$^{\mathrm{w}}$: for all $N \in \mathcal{N}$, $M \subset N, u, u' \in \mathbb{R}^N$, if $u_i = u'_i$ for all $i \in N \backslash M$, then

$$\begin{aligned} u\, \underline{R}\,(N)\; u' \;&\Rightarrow\; u_M\, \underline{R}\,(M)\; u'_M, \\ u\, \underline{P}\,(N)\; u' \;&\Rightarrow\; u_M\, \underline{P}\,(M)\; u'_M. \end{aligned}$$

If social welfare is defined as the sum (utilitarianism) or the product (the Nash social welfare function) or the leximin (the so-called Rawlsian social welfare function) of agents' utilities, then the resulting social welfare ordering function satisfies Consistency$^{\mathrm{w}}$. Generalized Gini social welfare orderings, on the other hand, do not satisfy this property (see, e. g., Blackorby, Bossert and Donaldson [2]).

The consistency property defined above looks similar to separability conditions that are encountered in welfare economics and the theory of social choice (see Fleming [11] and d'Aspremont and Gevers [8]). The separability conditions, with some variations, state that agents who are indifferent over some alternatives should not influence social preferences over those alternatives. Our condition says that removing those agents from the economy should not alter social preferences.

[2] As it does not satisfy Consistency, applying the Roemer proposal to the school funding system, for instance, will lead different results if, say, the relative funding of schools A and B are computed as part of a City or a State policy. This may be viewed as a weakness of that approach.

[3] Regarding the second branch of literature, Consistency is studied in e. g. Fleurbaey [13]. As of the third branch, Consistency is compatible with e. g. the cardinality based approach of Ok and Kranich [19].

4 The result

We are now equipped to prove our main result. It is an equivalence result between two seemingly unrelated equity theories. If an allocation ordering function is consistent with the goal of equalizing opportunities in the sense of satisfying properties of compensation and responsibility as stated in the previous section and satisfies Consistency, then it is welfarist. Conversely, given any welfarist way of aggregating utility levels, there exists a way of constructing utility functions such that the resulting ordering function equalizes opportunities in the sense above.

There are two ways of stating this result, depending on which of the equal opportunity requirements we emphasize (recall that the existence of allocation ordering functions satisfying Compensation and Responsibility is not guaranteed in our model). Indeed, we can either combine Compensation and Minimal Responsibility, or Minimal Compensation and Responsibility. This gives us the following theorems.

Theorem 1 *Under Assumption A, if an allocation ordering function $\overline{R}$ satisfies* Social Indifference*,* Compensation*,* Minimal Responsibility *and* Consistency*, then it is* Welfarist *and the real vector ordering function $\underline{R}$ associated to it satisfies* Anonymity^w *and* Consistency^w.

Theorem 2 *For each real vector ordering function $\underline{R}$ satisfying* Anonymity$^{\mathrm{w}}$*, there exists a* Welfarist *allocation ordering function $\overline{R}$ satisfying* Social Indifference*,* Compensation *and* Minimal Responsibility.

We give the proof of those statements in Section 6. Let us, here, explain in words why equality of opportunity turns out to be equivalent to welfarism. Let us start with any given economy. It is well-known in this case that any allocation ordering satisfying Social Indifference can be represented as a social welfare ordering (see e. g. Blackorby, Donaldson and Weymark [4]). This is called in the literature single profile welfarism, and it is a well-known fact that if parameters are not allowed to change, that is, if the ethical observer is not required to be consistent in her judgement over different economies, then she is always a welfarist. This first step is no more than a technical representation and it has nothing to do with the requirement that social welfare orderings be independent of the particular utility functions that create the utility levels.

But the key feature of the equality of opportunity properties is that they allow us to extend this single profile welfarism into welfarism. This comes from the fact that equality of opportunity combines two orthogonal requirements, one on compensation parameters and the other one on responsibility parameters. Moreover, *Consistency* imposes robustness across orderings in different economies, so that the restriction on the ordering in one society contaminates the orderings in the others.

Let us illustrate these facts. Let us consider a two-agent economy $(\{1,2\}, (\theta_{c1},\theta_{c2}), (\theta_{r1},\theta_{r2}))$ where individual characteristics are different. None of our compensation and responsibility properties seem to have any bite on this economy. However, our social evaluation need to be consistent with, say, economy $(\{1,2,3\}, (\theta_{c1},\theta_{c2},\theta_{c3}), (\theta_{r1},\theta_{r2},\theta_{r3}))$ where we can assume that $\theta_{c3} = \theta_{c1}$ and $\theta_{r3} = \theta_{r2}$. In that economy, a compensation property imposes restriction on the pair $\{2,3\}$, and a responsibility property on the pair $\{1,3\}$. Moreover, the social evaluation in the initial economy also needs to be consistent with that in economy $(\{1,2,4\}, (\theta_{c1},\theta_{c2},\theta_{c4}), (\theta_{r1},\theta_{r2},\theta_{r4}))$ where $\theta_{c4} = \theta_{c2}$ and $\theta_{r4} = \theta_{r1}$. Now, compensation imposes a restriction on the pair $\{1,4\}$ and responsibility on the pair $\{2,4\}$. As it turns out, those cross economies restrictions are so severe that once the allocation ordering is fixed for one arbitrary economy, there is only one consistent ordering for any other economy, thereby extending single profile welfarism into welfarism. The two versions of the result simply states that combining Compensation and Minimal Responsibility or Minimal Compensation and Responsibility respectively is sufficient to determine the required initial allocation ordering in some specific economy.

Theorem 3 *Under Assumption A, if an allocation ordering function* $\overline{R}$ *satisfies* Social Indifference*,* Minimal Compensation*,* Responsibility *and* Consistency*, then it is* Welfarist *and the real vector ordering function* $\underline{R}$ *associated to it satisfies* Anonymity *and* Consistency^w.

Theorem 4 *For each real vector ordering function* $\underline{R}$ *satisfying* Anonymity^w*, there exists a* Welfarist *allocation ordering function* $\overline{R}$ *satisfying* Social Indifference*,* Minimal Compensation *and* Responsibility.

The proof of the last theorems parallels the proof of the previous ones and will be omitted.

5 Ordinalism non-comparability

We now turn to the question of the informational basis of the ethical theories we have been studying. Which measurability and comparability assumption do we have to impose on preferences to be able to construct equal opportunity or welfarist social ordering functions? We prove in this Section that all the results that have been stated up to now are compatible with the weakest informational assumptions on preferences. Indeed, constructing utility functions in a way consistent with Theorems 1 and 3 is possible even with only ordinal non-comparable information on preferences, provided the parameters determining agents' utility functions are part of the responsibility parameters. The latter proviso is advocated by some political philosophers (see Rawls [20], Dworkin [10]).

Throughout this section, we consider that outcomes are actually utility levels. That is, we fix $\mathcal{O} \subseteq \mathbb{R}$ (recall that $\mathcal{O}$ is the image set of outcome function) so that $O(x_i, \theta_{ic}, \theta_{ir})$ stands for the utility level reached by an agent i having parameters θ_{ic}, θ_{ir} and being assigned resources x_i. Society considers that agents are responsible for their utility function, that is, what determines agents' utility functions is part of their responsibility parameters. For the sake of simplicity, we will assume that the responsibility parameters gather all and only what makes utility functions differ from one agent to another. We will come back to this assumption at the end of the section.

A resource allocation ordering function satisfies *Ordinalism Non-Comparability* whenever the rankings of the allocations only depend on the individual rankings and not on the utility levels they reach. That is, if the utility function of an agent changes so that her new function is simply obtained by a strictly increasing transformation of her first one, then the social ranking should not change.

Ordinalism Non-Comparability: for all $e = (N, \theta_c, \theta_r), \in \mathcal{E}$, $j \in N, \theta'_{rj} \in \Theta_r$ if there exists a strictly increasing function $g : \mathbb{R} \to \mathbb{R}$, such that for all $x \in X$,

$$O(x, \theta_{cj}, \theta_{rj}) = g\left(O(x, \theta_{cj}, \theta'_{rj})\right)$$

then

$$\overline{R}(e) = \overline{R}(N, \theta_c, (\theta_{rN\setminus\{j\}}, \theta'_{rj})).$$

Our second result is that the properties required in the previous section are compatible with *Ordinalism Non-Comparability.*

Theorem 5 *If an allocation ordering function* $\overline{R}$ *satisfies* Responsibility *and* Consistency, *then it satisfies* Ordinalism Non-Comparability.

If *Responsibility* is replaced with *Minimal Responsibility*, then we would only obtain *Ordinalism Non-Comparability* for agents having the reference parameter $\widetilde{\theta}_r$. Nonetheless, if *Compensation* is added, then, by the same kind of argument as in the previous section, we can widespread this result to all other responsibility parameters.

Let us come back to the assumption that the whole list of responsibility parameters determine agents' utility function. Let us now assume, instead, that there are two lists of parameters, respectively θ_r and θ_u whereas only θ_u enters the definition of the utility function. *Responsibility* would still be stated with the proviso that $\theta_{cj} = \theta_{ck}$, and *Ordinalism Non-Comparability* would be adapted so that the utility transformation reads

$$O(x, \theta_{cj}, \theta_{rj}, \theta_{uj}) = g\left(O(x, \theta_{cj}, \theta_{rj}, \theta'_{uj})\right).$$

It is clear that the statement just proven would hold a fortiori. So it is sufficient that the parameters determining the utility function be part of what society would like to hold agents responsible for.

6 Proofs

We begin this proof section by stating and proving an important consequence of the *Consistency* property.

Lemma 1 *If a social ordering function* $\overline{R}$ *satisfies* Consistency*, then it satisfies the following property: for all* $e = (N, \theta_c, \theta_r), e' = (M, \theta_{cM}, \theta_{rM}) \in \mathcal{E}$, *such that* $N \cap M = \emptyset$, $x, x' \in X^N$ *and* $y \in X^{|M|}$: $x\,\overline{R}(e)\,x'$ *if and only if* $(x, y)\,\overline{R}(e, e')\,(x', y)$.

Proof. Suppose the claim is wrong, so that, for instance, $x\,\overline{R}(e)\,x'$ whereas $(x', y)\,\overline{P}(e, e')\,(x, y)$ (a simlar argument works in the case $x\,\overline{P}(e)\,x'$ whereas $(x', y)\,\overline{I}(e, e')\,(x, y)$). By *Consistency*, $(x', y)\,\overline{P}(e, e')\,(x, y)$ implies $x'\,\overline{P}(e)\,x$, a contradiction which proves the claim. ■

6.1 Theorem 1

Proof. Let $\overline{R}$ satisfy the axioms. Let $\widetilde{\theta}_c$ be a compensation parameter value for which $\overline{R}$ satisfies *Minimal Responsibility*. Let us fix $N \in \mathcal{N}$. Let $\theta_r^* \in \Theta_r$ be any responsibility parameter value. Let us define the economy $\widetilde{e} = (N, (\widetilde{\theta}_c, \ldots, \widetilde{\theta}_c), (\theta_r^*, \ldots, \theta_r^*)) \in \mathcal{N}$. By *Social Indifference*, we can use Propositions 1 and 2 in Blackorby, Donaldson and Weymark [4], so that there exist a social welfare ordering, say $\underline{R}$, on $\mathbb{R}^N$, and a utility function, say $u \in \mathcal{U}$ such that for all $x, x' \in X^N$,

$$x \, \overline{R}(\widetilde{e}) \, x' \Leftrightarrow u \, \underline{R} \, u'$$

where for all $i \in N$, $u_i = u(x_i)$ and $u'_i = u(x'_i)$ and u represents O at $\widetilde{\theta}_c, \theta_r^*$, that is,

$$[O(x, \widetilde{\theta}_c, \theta_r^*) \geq O(x', \widetilde{\theta}_c, \theta_r^*)] \Leftrightarrow [u(x) \geq u(x')].$$

By *Compensation*, for all $\pi \in \Pi_N$, $x \, \overline{I}(\widetilde{e}) \, \pi(x)$ and $x' \, \overline{I}(\widetilde{e}) \, \pi(x')$. Therefore, $u \, \underline{R} \, u' \Leftrightarrow \pi(u) \, \underline{R} \, \pi(u')$, and $\underline{R}$ satisfies *Anonymity*$^{\mathrm{w}}$.

Step 1: construction of $u_{..}$.

For all $x \in X$, let $u_{\widetilde{\theta}_c \theta_r}(x) \equiv u(x)$. For all $\theta_c \in \Theta_c$, by Assumption A, for all $\widetilde{x} \in X$, there exists $x \in X$ such that $O(x, \theta_c, \theta_r) = O(\widetilde{x}, \widetilde{\theta}_c, \theta_r)$. Let u_{θ_c, θ_r} be defined by: for all $x \in X$,

$$u_{\theta_c, \theta_r}(x) = u(\widetilde{x}) \Leftrightarrow O(x, \theta_c, \theta_r) = O(\widetilde{x}, \widetilde{\theta}_c, \theta_r).$$

Finally, for all $\theta'_r \in \Theta_r$, $u_{\theta'_c, \theta_r}$ be defined by: for all $x \in X$,

$$u_{\theta_c, \theta'_r}(x) = u_{\theta_c, \theta_r}(x).$$

Step 2: $\overline{R}$ is *Welfarist*

Let $e = (N, \theta_{cN}, \theta_{rN}) \in \mathcal{N}, x, x' \in X^N$. We have to show that

$$x \, \overline{R}(e) \, x' \Leftrightarrow u \, \underline{R} \, u'$$

where for all $i \in N$, $u_i = u_{\theta_{ic}, \theta_{ir}}(x_i)$ and $u'_i = u_{\theta_{ic}, \theta_{ir}}(x'_i)$. Let $N' \in \mathcal{N}$ be such that $|N| = |N'|$ so that there exists a bijection $\beta : N \to N'$. Let $e' = (N', (\widetilde{\theta}_c, \ldots, \widetilde{\theta}_c), \theta'_r) \in \mathcal{E}, y, y' \in X^{N'}$ be such that for all $i \in N, \theta_{ri} = \theta'_{r\beta(i)}$, $u_{\widetilde{\theta}_c, \theta'_{r\beta(i)}}(y_{\beta(i)}) = u_i$ and $u_{\widetilde{\theta}_c, \theta'_{r\beta(i)}}(y'_{\beta(i)}) = u'_i$. By *Consistency* and Lemma 1,

$$x \, \overline{R}(e) \, x' \Leftrightarrow (x, y') \, \overline{R}(e, e') \, (x', y').$$

By *Compensation* applied to every pair of agents $i, \beta(i)$, $(x, y')\,\overline{I}(e, e')\,(x', y')$. Therefore,

$$(x, y')\,\overline{R}(e, e')\,(x', y') \Leftrightarrow (x', y)\,\overline{R}(e, e')\,(x', y').$$

By *Consistency*,

$$(x', y)\,\overline{R}(e, e')\,(x', y') \Leftrightarrow y\,\overline{R}(e')\,y'.$$

Recall that $\widetilde{e} = (N, (\widetilde{\theta}_c, \ldots, \widetilde{\theta}_c), (\theta_r^*, \ldots, \theta_r^*))$. Let $\widetilde{x}, \widetilde{x}' \in X^N$ be defined by for all $i \in N$, $\widetilde{x}_i = y_{\beta(i)}$ and $\widetilde{x}'_i = y'_{\beta(i)}$. By *Consistency* and Lemma 1,

$$y\,\overline{R}(e')\,y' \Leftrightarrow (y, \widetilde{x}')\,\overline{R}(e', \widetilde{e})\,(y', \widetilde{x}').$$

By *Minimal Responsibility*, $(y, \widetilde{x}')\,\overline{I}(e', \widetilde{e})\,(y', \widetilde{x}')$. Therefore,

$$(y, \widetilde{x}')\,\overline{R}(e', \widetilde{e})\,(y', \widetilde{x}') \Leftrightarrow (y', \widetilde{x})\,\overline{R}(e', \widetilde{e})\,(y', \widetilde{x}').$$

By *Consistency*,

$$(y', \widetilde{x})\,\overline{R}(e', \widetilde{e})\,(y', \widetilde{x}') \Leftrightarrow \widetilde{x}\,\overline{R}(\widetilde{e})\,\widetilde{x}'.$$

To sum up,

$$x\,\overline{R}(e)\,x' \Leftrightarrow \widetilde{x}\,\overline{R}(\widetilde{e})\,\widetilde{x}'.$$

Now, we know that

$$\widetilde{x}\,\overline{R}(\widetilde{e})\,\widetilde{x}' \Leftrightarrow \widetilde{u}\,\underline{R}\,\widetilde{u}'$$

where for all $i \in N$, $\widetilde{u}_i = u_{\widetilde{\theta}_{ic}, \theta_{ir}^*}(\widetilde{x}_i)$ and $\widetilde{u}'_i = u_{\widetilde{\theta}_{ic}, \theta_{ir}^*}(\widetilde{x}'_i)$. By construction, for all $i \in N$, $u_{\theta_{ic}, \theta_{ir}}(x_i) = u_{\widetilde{\theta}_{ic}, \theta_{ir}^*}(\widetilde{x}_i)$ and $u_{\theta_{ic}, \theta_{ir}}(x'_i) = u_{\widetilde{\theta}_{ic}, \theta_{ir}^*}(\widetilde{x}'_i)$. Therefore,

$$\widetilde{u}\,\underline{R}\,\widetilde{u}' \Leftrightarrow u\,\underline{R}\,u'.$$

Combining all the equivalences, we get, by transitivity,

$$x\,\overline{R}(e)\,x' \Leftrightarrow u\,\underline{R}\,u',$$

the desired outcome. ■

6.2 Theorem 2

Proof. Let us fix some $\widetilde{\theta}_c \in \Theta_c$. Let $u_{..}$ be some utility representation of O satisfying the following properties: for all $\theta_c \in \Theta_c, \theta_r, \theta'_r \in \Theta_r, x, x' \in X$,

$$\begin{aligned} u_{\theta_c, \theta_r}(x) &= u_{\theta_c, \theta'_r}(x) \\ u_{\theta_c, \theta_r}(x) = u_{\widetilde{\theta}_c, \theta_r}(x') &\Leftrightarrow O(x, \theta_c, \theta_r) = O(x', \widetilde{\theta}_c, \theta_r). \end{aligned}$$

Let $\overline{R}$ be defined by for all $e = (N, \theta_c, \theta_r) \in \mathcal{E}$, $x, x' \in X^N$

$$x \, \overline{R}(e) \, x' \Leftrightarrow u \, \underline{R}(e) \, u'$$

where for all $i \in N$, $u_i = u_{\theta_{ic}, \theta_{ir}}(x_i)$ and $u'_i = u_{\theta_{ic}, \theta_{ir}}(x'_i)$. By construction, $\overline{R}$ is *Welfarist*. We prove that it satisfies the axioms.
1) *Social Indifference*: It comes directly from the fact that, by *Welfarism*, $\overline{R}$ does not discriminate between allocations yielding the same utility vector.
2) *Compensation*: Let us take any $e = (N, \theta_c, \theta_r) \in \mathcal{E}$, $j, k \in N$ such that $\theta_{rj} = \theta_{rk}$, $x, x' \in X^N$ such that $x_i = x'_i$ for all $i \neq j, k$ and $O(x_j, \theta_{cj}, \theta_{rj}) = O(x'_k, \theta_{ck}, \theta_{rk})$ and $O(x'_j, \theta_{cj}, \theta_{rj}) = O(x_k, \theta_{ck}, \theta_{rk})$. By *Anonymity*$^{\mathrm{w}}$,

$$u \, \underline{I}(N) \, u',$$

where for all $i \in N$, $u_i = u_{\theta_{ic}, \theta_{ir}}(x_i)$ and $u'_i = u_{\theta_{ic}, \theta_{ir}}(x'_i)$. By construction of $\overline{R}$,

$$x \, \overline{I}(e) \, x'.$$

3) *Minimal Responsibility*: Let us take any $e = (N, \theta_c, \theta_r) \in \mathcal{E}$, $j, k \in N$ such that $\theta_{cj} = \theta_{ck} = \tilde{\theta}_c$, and any $x, x' \in X^N$ such that $x_i = x'_i$ for all $i \neq j, k$ and $x_j = x'_k$ and $x'_j = x_k$. By construction of $u_{.,.}$, $u_{\tilde{\theta}_c, \theta_{ri}}(x) = u_{\tilde{\theta}_c, \theta_{rj}}(x)$ and $u_{\tilde{\theta}_c, \theta_{ri}}(x') = u_{\tilde{\theta}_c, \theta_{rj}}(x')$. By *Anonymity* of $\underline{R}$,

$$u \, \underline{I}(N) \, u'$$

where for all $i \in N$, $u_i = u_{\theta_{ic}, \theta_{ir}}(x_i)$ and $u'_i = u_{\theta_{ic}, \theta_{ir}}(x'_i)$. By construction of $\overline{R}$,

$$x \, \overline{I}(e) \, x',$$

the desired outcome. ■

6.3 Theorem 5

We first need an additional definition and a lemma. *Anonymity* requires that the names of the agents do not matter in social judgements, that is, if two agents permute their parameters, then the social ranking remains unaffected provided the resources allocated to those agents are also permuted.

Anonymity: for all $e = (N, \theta_c, \theta_r), \in \mathcal{E}$, $j, k \in N$, $x, x' \in X^N$, $(\theta'_{cj}, \theta'_{rj})$, $(\theta'_{ck}, \theta'_{rk}) \in \Theta_c \times \Theta_r$, such that $(\theta'_{cj}, \theta'_{rj}) = (\theta_{ck}, \theta_{rk})$ and $(\theta'_{ck}, \theta'_{rk}) = (\theta_{cj}, \theta_{rj})$, and $x_j'', x_k'', x_j''', x_k'''$ such that $x_j'' = x_k, x_k'' = x_j, x_j''' = x_k'$ and $x_k''' = x_j'$,

$$\begin{aligned} x\,\overline{R}(e)\,x' &\Leftrightarrow \\ \left(x_{N\backslash\{j,k\}}, x_j'', x_k''\right) &\overline{R}(\quad e')\left(x_{N\backslash\{j,k\}}', x_j''', x_k'''\right), \end{aligned}$$

where $e' = (N, (\theta_{cN\backslash\{j,k\}}, \theta_{cj}', \theta_{ck}'), (\theta_{rN\backslash\{j,k\}}, \theta_{rj}', \theta_{rk}'))$.

Lemma 2 *If an allocation ordering function $\overline{R}$ satisfies* Responsibility, *and* Consistency, *then it satisfies* Anonymity.

Proof. Let $e = (N, \theta_c, \theta_r), \in \mathcal{E}, x, x' \in X^N$ be such that

$$x\,\overline{R}(e)\,x'.$$

Let $j, k \in N$. Let $\{l, m\} \subset \mathcal{N}\backslash N$. Let $\theta_j', \theta_k', \theta_l, \theta_m \in \Theta_c \times \Theta_r$ and x_j'', x_k'', x_j''', x_k''', x_l, x_m, x_l', x_m' be defined by

$$\begin{aligned} \theta_j' = \theta_m &\equiv \theta_k, \\ \theta_k' = \theta_l &\equiv \theta_j, \\ x_j'' = x_m &\equiv x_k, \\ x_k'' = x_l &\equiv x_j, \\ x_j''' = x_m' &\equiv x_k', \text{ and} \\ x_k''' = x_l' &\equiv x_j'. \end{aligned}$$

For the sake of convenience, let us define $M = N\backslash\{j,k\}, N' = N\backslash\{j,k\} \cup \{l,m\}$, and $P = N \cup \{l,m\}$. By *Consistency*,

$$(x, x_l', x_m')\,\overline{R}\left(P, (\theta_c, \theta_{cl}, \theta_{cm}), (\theta_r, \theta_{rl}, \theta_{rm})\right)(x', x_l', x_m').$$

By *Responsibility*,

$$\left(x_M, x_j', x_k', x_l, x_m\right)\overline{R}\left(P, (\theta_c, \theta_{cl}, \theta_{cm}), (\theta_r, \theta_{rl}, \theta_{rm})\right)(x', x_l', x_m').$$

By *Consistency*,

$$(x_M, x_l, x_m)\,\overline{R}\left(N', (\theta_{cM}, \theta_{cl}, \theta_{cm}), (\theta_{rM}, \theta_{rl}, \theta_{rm})\right)(x_M', x_l', x_m').$$

Let $\theta_{cP}^* = \left(\theta_{cM}, \theta_{cj}', \theta_{ck}', \theta_{cl}, \theta_{cm}\right)$ and $\theta_{rP}^* = \left(\theta_{rM}, \theta_{rj}', \theta_{rk}', \theta_{rl}, \theta_{rm}\right)$. By *Consistency* again,

$$\left(x_M, x_j'', x_k'', x_l, x_m\right)\overline{R}\left(P, \theta_{cP}^*, \theta_{rP}^*\right)\left(x_M', x_j'', x_k'', x_l', x_m'\right).$$

By *Responsibility*,

$$\left(x_M, x''_j, x''_k, x_l, x_m\right) \overline{R}\left(P, \theta^*_{cP}, \theta^*_{rP}\right) \left(x'_M, x'''_j, x'''_k, x_l, x_m\right).$$

By *Consistency*,

$$\left(x_M, x''_j, x''_k\right) \overline{R}\left(N, \left(\theta_{cM}, \theta'_{cj}, \theta'_{ck}\right), \left(\theta_{rM}, \theta'_{rj}, \theta'_{rk}\right)\right) \left(x'_M, x'''_j, x'''_k\right),$$

the desired outcome. ■

Proof. (of Theorem 5.1) By Lemma 2, $\overline{R}$ satisfies *Anonymity*. Let $e = (N, \theta_c, \theta_r), \in \mathcal{E}$, $j \in N, \theta'_{rj} \in \Theta_r$, be such that there exists a strictly increasing function $g : \mathbb{R} \to \mathbb{R}$, such that for all $x \in X$,

$$O(x, \theta_{cj}, \theta_{rj}) = g\left(O(x, \theta_{cj}, \theta'_{rj})\right).$$

Let $x, x' \in X^N$ be such that

$$x\,\overline{R}(e)\,x'.$$

We need to prove that

$$x\,\overline{R}(N, \theta_c, (\theta_{rN\setminus\{j\}}, \theta'_{rj}))\,x'.$$

Let $k \notin N, \theta_{ck} \in \Theta_c, \theta_{rk}, \theta'_{rk} \in \Theta_r, x_k, x'_k \in X$ be such that $\theta_{ck} = \theta_{cj}$,$\theta_{rk} = \theta_{rj}, \theta'_{rk} = \theta'_{rj}, x_k = x_j$ and $x'_k = x'_j$. By *Consistency* and Lemma 1,

$$(x, x'_k)\ \overline{R}\left(N \cup \{k\}, (\theta_c, \theta_{ck}), (\theta_r, \theta'_{rk})\right)\ (x', x'_k).$$

By *Anonymity*,

$$\left(x_{N\setminus\{j,k\}}, x'_j, x_k\right)\ \overline{R}\left(N \cup \{k\}, (\theta_c, \theta_{ck}), (\theta_{rN\setminus\{j\}}, \theta'_{rj}, \theta_{rk})\right)\ (x', x'_k).$$

Since $\theta_{ck} = \theta_{cj}$, by *Responsibility*,

$$\left(x_{N\setminus\{j,k\}}, x_j, x'_k\right)\ \overline{R}\left(N \cup \{k\}, (\theta_c, \theta_{ck}), (\theta_{rN\setminus\{j\}}, \theta'_{rj}, \theta_{rk})\right)\ (x', x'_k).$$

By *Consistency*,

$$x\,\overline{R}(N, \theta_c, (\theta_{rN\setminus\{j\}}, \theta'_{rj}))\,x',$$

the desired outcome. ■

7 Conclusion

We have shown in this paper that there is an equivalence between two seemingly unrelated, if not opposed, families of resource allocation ordering functions. The first family of functions, which we consider consistent with modern equal opportunity theories, satisfy properties of compensation and responsibility. The second family of functions are the welfarist functions. We have shown that among the ordering functions satisfying *Consistency*, each member of the first family was also a member of the second one, and vice versa.

We would like to emphasize again that allocation ordering functions as defined here can be constructed without making use of cardinal or comparable information on preferences. This comes from the fact that the welfarist ordering functions we deal with here, are not based on the assumption that utility functions are intrinsically related to the human nature. Instead, utility functions are assumed to be constructed by social observers, and, precisely, the observers we were interested in are equal opportunity advocates. This is why it is possible to be welfarist and still use only ordinal non-comparable information on individual preferences.

There are several consequences to draw from our results. Maybe the main lesson is that welfarism is far from being an obsolete ethical theory. Moreover, welfare economists can do more than letting the social planner construct the utility respresentation of agents' preferences. Indeed, the social planner should only be asked to choose the cut between compensation and responsibility parameters, and the aggregation process of individual opportunity indicators. Welfare economists should then be able to compute, for instance, optimal tax shemes, in a similar way as it is currently done in public economics. If, in addition, the social planner considers individual preferences as part of the responsibility parameters, then all the information needed to compute the optimal allocation can be extracted from agents' choices. The results persented here should therefore yield new approaches to the design of optimal social policies.

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